## Indian Statistical Institute, Bangalore Centre B.Math. (III Year) : 2009-2010 Semester I : Backpaper Examination Introduction to Differential Equations

Time: 3 hours

Maximum Marks: 100

1. [10 marks] Suppose a radioactive material disintegrates at a rate proportional to the amount of material present. Take the proportionality factor to be a known constant. Find the time required for the mass to be reduced to one half of its original value.

2. [12 marks] Find the general solution to

$$x''(t) - 4x(t) + 3 = 0, \quad t > 0$$

3. [15 marks] Solve the problem:

$$x''(t) - 2x'(t) = 12t - 10, \ t > 0, \ x(0) = 0, x'(0) = 0.$$

4. [16 marks] Consider Chebyshev's ODE

$$(1-t^2)x''(t) - tx'(t) + \lambda^2 x(t) = 0, \quad |t| < 1,$$

where  $\lambda$  is a real constant. Find two linearly independent solutions by method of power series. If  $\lambda$  is a nonnegative integer show that there is a polynomial solving the equation.

5. [15 marks] Find  $u(\cdot, \cdot)$  satisfying

$$u_t(t,x) + 2u_x(t,x) + 3u(t,x) = 0, t > 0, x \in \mathbb{R}$$

with  $u(0, x) = x^2$ .

6. [12 marks] Find the solution to the Dirichlet problem

$$\Delta u(r,\phi) = 0, \quad 1 < r < 2, -\pi \le \phi < \pi$$

with  $u(1,\phi) = 3, u(2,\phi) = 8, -\pi \le \phi < \pi.$ 

7. [20 marks] Let  $f(\cdot), g(\cdot), \alpha(\cdot), \beta(\cdot)$  be linear functions on the real line. Solve

$$u_{tt}(t,x) = u_{xx}(t,x) + \beta(t)\alpha(x), \quad t > 0, x \in \mathbb{R}$$

with  $u(0,x) = f(x), u_t(0,x) = g(x), x \in \mathbb{R}$ .